It follows from the above results that when thermomechanical coupling is taken into account a viscoelastic rod behaves as a nonlinear mechanical system with a soft type of characteristic.

NOTATION

x	is the coordinate along the rod;
l	is the length of the rod;
c_2, c_2, β, γ, T_1	are the constants of the material;
T	is the temperature;
$\sigma = \sigma_1 + i\sigma_2$	is the complex amplitude of the harmonic stress;
ρ	is the density;
λ_2	is the thermal conductivity;
ω	is the angular frequency;
ξ, θ, s_1, s_2	are the dimensionless coordinate, temperature, and stress components;
λ	is the dimensionless loading parameter.

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CALCULATION OF KINEMATIC COAGULATION OF AN AEROSOL IN A VARIABLE-SPEED GAS STREAM

I. B. Palatnik and A. K. Azhibekov

UDC 532.529.6

A method is suggested for calculating the kinematic coagulation of drops in a variable-speed gas stream when they are broken up by the gas stream. The results of the calculation are compared with test data.

The problem of coagulation, particularly of colloids, under the action of Brownian motion was first analyzed by Smolukhovskii [1] for the case of an isodisperse distribution. The equations for the general case of coagulation with a continuous polydisperse distribution were analyzed by Müller [2] and Tunitskii [3].

Two approaches to the calculation of particle coagulation are known (see [4], for example). The first is based on the study of the evolution of the drop sizes of the fractions under consideration. This method, because of a certain analogy with classical hydrodynamics, received the name of the Lagrange method. The second is based on the determination of the numbers of particles of fixed sizes and is named the Euler method.

Henceforth we will analyze the problem of coagulation of a polydisperse system of particles by the Euler method.

When the particle spectrum is assigned in the form of a varying mass distribution function

$$dN = f(m, \tau)\, dm, \tag{1}$$

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where dN is the number of particles in the interval dm, $f(m, \tau)$ is the computed density distribution, and τ is the time, the time variation of the function f is described, according to [2, 3], by the equation

$$\frac{df(m, \tau)}{d\tau} = I_1(m, \tau) - I_2(m, \tau), \tag{2}$$

where

$$I_1(m, \tau) = \frac{1}{2}\int_0^m K(m', m-m', \tau) f(m', \tau) f(m-m', \tau)\, dm'. \tag{2a}$$

$$I_2(m, \tau) = f(m, \tau)\int_0^\infty K(m^*, m, \tau) f(m^*, \tau)\, dm^*. \tag{2b}$$

In Eqs. (2)-(2b), K is the coefficient of particle coagulation, determined by the mechanism of occurrence of the process; I_1 and I_2 are integrals expressing the number of particles of mass m forming in the collision of particles having masses m' and m − m' and the number of particles of mass m lost upon their collisions with any particles of mass m^*, coalescence, and leaving the category of particles of the given mass.

Using the calculating particle size distribution function

$$dN = f(\delta, \tau)\, d\delta \tag{3}$$

the coagulation equation for the function $f(\delta, \tau)$ was obtained in [5] in a form analogous to (2), with the difference that

$$I_1(\delta, \tau) = \int_0^{\delta/\sqrt[3]{2}} K(\delta', \sqrt[3]{\delta^3-(\delta')^3}, \tau) f(\delta', \tau) f(\sqrt[3]{\delta^3-(\delta')^3}, \tau)\, \varphi(\delta, \delta')\, d\delta', \tag{2c}$$

$$I_2(\delta, \tau) = f(\delta, \tau)\int_0^\infty K(\delta^*, \delta, \tau) f(\delta^*, \tau)\, d\delta^*. \tag{2d}$$

The function

$$\varphi(\delta, \delta') = 1/[1-(\delta'/\delta)^3]^{2/3} \tag{4}$$

was introduced into (2c). Its introduction is connected with the fact that, because of the nonlinear identity $\delta^3 = (\delta')^3 + (\sqrt[3]{\delta^3-(\delta')^3})^3$, the widths of the intervals in (2c) for particles with sizes δ, δ', and $\sqrt[3]{\delta^3-(\delta')^3}$ prove to be unequal to each other (see [5, 6]).

For the motion of a coagulating aerosol in a variable-speed gas stream in a channel of variable cross section a correction was introduced in [5] allowing for the fact that the calculating particle size distribution density $f(\delta, \tau)$ varies not only due to coagulation but also in connection with the fact that the velocities of the particles and the gas not are equal to each other.

Using the continuity equation for the calculating distribution density function for particles of a given fraction, we obtain

$$f(\delta, \tau_0)\, u(\delta, \tau_0)\, s(\tau_0) = f(\delta, \tau)\, u(\delta, \tau)\, s(\tau) = \text{const}, \tag{5}$$

where u and s are the particle velocity and the channel cross section, respectively.

Using (5), the particle coagulation equation for the calculating size distribution function can be written in the form

$$f(\delta, \tau) = \frac{u(\delta, \tau_0)\, s(\tau_0)}{u(\delta, \tau)\, s(\tau)}\left[f(\delta, \tau_0) + \int_0^\tau (I_1(\delta, \tau) - I_2(\delta, \tau)\, d\tau)\right]. \tag{6}$$

As shown in [5], in a number of cases Eq. (6) describes the observed test data quite satisfactorily.

But in the general case, such as when the spectrum of the particle size distribution is very broad and the coagulation takes place with considerable variation of the velocity and the channel cross section, the problem under consideration requires a more rigorous approach, in our opinion.

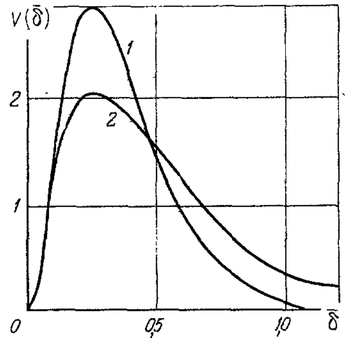

Fig. 1

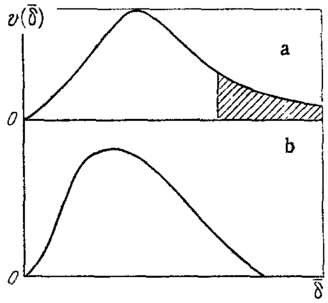

Fig. 2

Fig. 1. Calculation of variation of bulk size distribution density of the flux of water drops along a Venturi tube: 1) bulk distribution density of flux of drops V (1/sec) by size $\bar{\delta} = \delta/\delta_c$ in the throat; 2) the same at the end of the Venturi tube.

Fig. 2. Bulk distribution density v ($1/m^3$) of water drops by size $\bar{\delta}$ (initial spectrum).

We introduce a varying calculating distribution function for the flux of particles with sizes from δ to δ + dδ:

$$d\Phi(\delta, \tau) = F(\delta, \tau)\, d\delta, \tag{7}$$

where $F(\delta, \tau) = f(\delta, \tau)u(\delta, \tau)s(\tau)$ is the particle flux density. The magnitude of the flux (or the flux density) for particles of a given size can vary only through coagulation and does not depend on the variation in the velocity and cross section of the gas stream. In this connection, Eq. (2) [when using Eqs. (2c)-(2d) in calculating the particle-size distribution] must be written in the following form for the variation of the particle flux density, in our opinion, in the case of variation of the velocity and cross section of the gas stream:

$$\frac{dF(\delta, \tau)}{d\tau} = u(\delta, \tau)\, s(\tau)\, [I_1(\delta, \tau) - I_2(\delta, \tau)], \tag{8}$$

where J_1 and J_2 are determined by (2c) and (2d).

By integrating (8) in the limits from $\tau = 0$ to τ we obtain

$$F(\delta, \tau) = F(\delta, \tau_0) + \int_0^{\tau} u(\delta, \tau)\, s(\tau)\, [I_1(\delta, \tau) - I_2(\delta, \tau)]\, d\tau. \tag{9}$$

Converting to the calculating distribution density function, after simple transformations we obtain from (9)

$$f(\delta, \tau) = \frac{u(\delta, \tau_0)\, s(\tau_0)}{u(\delta, \tau)\, s(\tau)} \left[f(\delta, \tau_0) + \int_0^{\tau} \frac{u(\delta, \tau)\, s(\tau)}{u(\delta, \tau_0)\, s(\tau_0)}\, [I_1(\delta, \tau) - I_2(\delta, \tau)\, d\tau] \right]. \tag{10}$$

Equation (10) allows for the variation of the calculating particle distribution density both due to coagulation and due to the variation of the velocity of the gas stream and the cross section. The physical meaning of (10) is that to determine the actual calculating particle concentration density at any time one reduces the variation in the number of particles due to coagulation (the difference $I_1(\delta, \tau) - I_2 \times (\delta, \tau)$ to the initial conditions of flow of the aerosol by the factor $u(\delta, \tau)s(\tau)/u(\delta, \tau_0)s(\tau_0)$, adds it to the initial number of particles, and then reduces it to the conditions of flow at the time τ by the factor $u(\delta, \tau_0)s(\tau_0)/u(\delta, \tau)s(\tau)$.

A rather typical example of the simultaneous occurrence of coagulation and a considerable variation in the stream velocity and cross section is the flow of an aerosol in a Venturi tube – a device widely used at present, particularly for the coagulation of dust particles with drops of atomized liquid and dust collection both for protection of the surrounding medium and for the removal of valuable products. The process of coagulation of water drops, which determines the resistance of the apparatus and the heat exchange in it [5], as well as the efficiency of the subsequent inertial (as a rule) precipitation of the drops together with the trapped dust, is also very important in this connection.

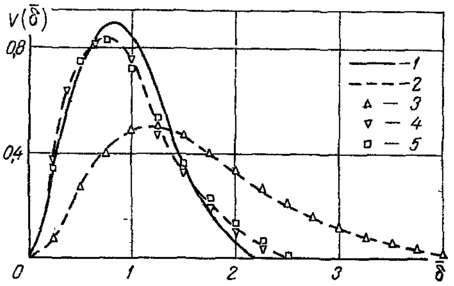

Fig. 3

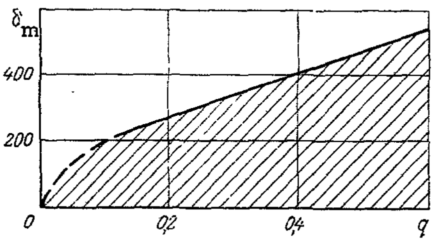

Fig. 4

Fig. 3. Comparison of calculated and experimental values of bulk distribution density of drop flux: 1) calculation by Eq. (10) (spectra in throat and at end of Venturi tube); 2) experiment; 3) initial spectrum; 4) spectrum in throat; 5) spectrum at end of Venturi tube.

Fig. 4. Criterion for choosing specific water flow rate (liter/m^3) and initial drop size δ_m (μm) to allow for coagulation.

Equation (10) is a complicated integrodifferential equation, and we know of no general methods for its solution. A finite-difference method, realized on a computer, was used in making practical calculations. The results of a calculation [5] within the limits of variation of the parameters of the problem under consideration show the stability and convergence of this method.

An example of a calculation of the variation of the bulk-size distribution density of the flux of water drops along a Venturi tube is shown in Fig. 1. The specific water flow rate is q = 1 liter/m^3, the model size of the bulk size distribution density of drops at the start of the Venturi tube is δ_m = 100 μm, and the characteristic size is δ_c = 400 μm. The geometrical parameters of this device are characteristic for power engineering with wet ash recovery.

As seen from the figure, in the case of a fine enough initial size distribution of drops, when another important process, breaking up of drops by the gas stream, is practically absent, their significant coagulation is observed, which must be taken into account. We note (and this is very important) that, as estimates showed, the total water flow rate is conserved with a sufficient accuracy (3%), in our opinion, which indicates the applicability of the calculating method used under the given conditions.

In the presence of a process of breakup of drops by the gas stream owing to their loss of stability Eq. (10) is of practical interest.

The basic quantitative relationships of the process of breakup of drops under the conditions characteristic of the atomizing of water in wet ash removers with Venturi tubes in power engineering were obtained in [7-9]. The bulk size distribution density of the products of the breakup of a single drop can be described by the dependence

$$v(\bar{\delta}) = 11.44\,\bar{\delta}^{1.25}\exp(-5.05\,\bar{\delta}^{2.25}), \tag{11}$$

where $\bar{\delta} = \bar{\delta}/D_0$ (D_0 is the size of the initial drop which breaks up).

We note that, as indicated in many investigations (see [10], for example), in the indicated range of Weber numbers the realization of the breakup of drops has an irregular character. At the lower limit only single drops break up; then as the Weber number grows the fraction of drops of a given size which break up grows, reaching 50% at We = 15.5; finally, as the upper limit is approached, the fraction of drops breaking up approaches 100%. A joint analysis of the data obtained in [7] and [9] showed that the distribution of the fraction of drops breaking up in the range of $8 \le \mathrm{We} \le 23$ is satisfactorily described by a normal-law distribution function:

$$C = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\frac{\mathrm{We}-15.5}{\sigma}} \exp\left(-\frac{t^2}{2}\right) dt, \tag{12}$$

where C is the fraction of drops of a given size which break up. The standard deviation σ is 2.5. C = 0 at We ≤ 8 and C = 1 at We ≥ 23.

The breakup relationships obtained were obtained with the calculation of coagulation by Eq. (10) as follows. If at some time τ the relative velocity between the gas and the drops reaches such a level that the portion of drops down to some size D_0 are broken up, then the mass of water corresponding to the hatched part (Fig. 2a) must be distributed in the rest of the spectrum in accordance with the dependence (11) and (12). This leads to some arbitrary distribution (Fig. 2b) which, starting with this time, must be taken as the initial distribution for subsequent calculations of the coagulation.

For an experimental test of this system for calculating coagulation with allowance for the breakup of drops by the gas stream, we made an experimental determination of the transformation of drops atomized by a centrifugal sprayer when they moved in a Venturi tube on a BKZ-160-100F boiler (station No. 10) of the Alma-Ata thermoelectric power plant TÉTs-1. The drop spectrum was determined experimentally by the pulse-counting method in accordance with the procedure presented in [11]. We note that when the indicated procedure is used the detector (thin coaxial metal needles) records the number of drops passing per unit time, i.e., the flux of drops. These readings can be identified with the size distribution of drop concentration only when the velocities of the drops and the gas are equal, which was not taken into account in [11]. A similar approach to the determination of the size distribution of drops is described in [12].

The results of a comparison of the experimental and calculated determination of the transformation of the spectrum along a Venturi tube are presented in Fig. 3. As is seen, their satisfactory agreement occurs. In the calculation the bulk density distribution of the flux of drops and determined in the form

$$V(\bar{\delta}) = \frac{\pi}{6}\,\bar{\delta}^3 F(\bar{\delta},\ \tau). \tag{13}$$

In these tests and calculations it was discovered that the greatest change in the drop spectrum occurs in the section of gas acceleration, mainly owing to the breakup. In the section of gas deceleration (the diffusor of the Venturi tube) the coagulation and breakup are not great under the given conditions.

An estimate of the necessity of allowing for the coagulation of water drops is important in the practical use of the equations presented, since most of the computer time in the calculations (up to 80%) goes just to the calculation of the coagulation. In this connection, we made a series of calculations using Eqs. (9) and (10) in order to establish the limits of variation of the water flow rates and the initial drop sizes for which one must allow for coagulation when the parameters of the Venturi tubes are characteristic for engineering. That combination of water flow rate and initial drop sizes at which the model size of the bulk distribution in the spectrum of the drop flux varied by not more than 2-5% upon moving from the throat to the end of the diffusor of the Venturi tube served as the criterion.

The results of this calculation are presented in Fig. 4. The region of combinations of specific water flow rate and initial drop sizes for which one must allow for coagulation lies below the boundary curve (hatched zone). Above it coagulation is not great and it can be neglected in the calculations. As seen from the figure, in power engineering, where the specific water flow rates do not exceed 0.1-0.15 liter/m^3 while the modal value of the initial drop distribution lies in the range of 300-500 μm, coagulation can be ignored, as a rule. In other cases [13], however, when the specific water flow rates are 1-2 liter/m^3, allowance for the coagulation of water drops is necessary (see Fig. 1, for example).

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SINGULARTIES OF DYNAMIC PROCESSES PROCEEDING IN DEFORMABLE SOLIDS WITH THE FINITE RATE OF HEAT PROPAGATION TAKEN INTO ACCOUNT

R. H. Shvets and A. A. Lopat'ev

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The singularities of dynamic processes occurring in deformable solids at high frequencies are studied on the basis of interrelated equations of the generalized theory of thermoelasticity.

The investigation of thermoelastic phenomena in solids has recently often been conducted on the basis of a generalized dynamical theory of thermoelasticity [1-3] with the finite rate of heat propagation in the solid taken into account. In this case the energy equation is an equation of hyperbolic type whose utilization for small times in the domain of large gradients would afford the opportunity for a more accurate description of the temperature fields [4] and the temperature stresses [2]. Experimental results on the dissipation of a heat pulse in liquid helium at very low temperatures are explained by using the hyperbolic equation of heat conduction.

In this connection, it is expedient to study the singularities of the thermoelastic processes proceeding in deformable solids by using the generalized dynamical theory of thermoelasticity.

Let us consider an infinite isotropic space possessing a thermal resistance. The thermoelastic motion of the solid can be described by the system of equations [3]

$$\mu\Delta\mathbf{u} + (\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,\mathbf{u} - (3\lambda+2\mu)\,\alpha\,\mathrm{grad}\,(T-T_0) = \rho\,\frac{\partial^2\mathbf{u}}{\partial\tau^2}\,,$$
$$\varkappa\Delta T = \frac{\partial T}{\partial\tau} + \tau_0\,\frac{\partial^2 T}{\partial\tau^2} + \gamma_1\left(\frac{\partial e_{ii}}{\partial\tau} + \tau_0\,\frac{\partial^2 e_{ii}}{\partial\tau^2}\right), \tag{1}$$
$$\sigma_{ij} = \lambda e_{ii}\delta_{ij} + 2\mu e_{ij} - (3\lambda+2\mu)\,\alpha\,(T-T_0)\,\delta_{ij}\,,$$

where δ_{ij} is the Kronecker delta, Δ is the Laplace operator $\gamma_1 = (3\lambda + 2\mu)\alpha T_0/\rho c_E$.

To simplify the computations, we go over to the dimensionless variables

$$z_i = \frac{\omega^*}{c_1}\,x_i\,,\ \tau_1 = \omega^*\tau,\ \mathbf{u}_1 = \frac{\lambda+2\mu}{3\lambda+2\mu}\,\frac{\omega^*}{c_1}\,\frac{\mathbf{u}}{\alpha T_0}\,,\ \theta = \frac{T-T_0}{T_0}\,,$$
$$\Sigma_{ij} = \frac{\sigma_{ij}}{(3\lambda+2\mu)\,\alpha T_0}\,,\ \gamma = \gamma_1\alpha\,\frac{3\lambda+2\mu}{\lambda+2\mu}\,,\ \beta = \frac{c_1^2}{c_q^2}\,, \tag{2}$$

in which system (1) becomes

$$\frac{\mu}{\lambda+2\mu}\,\Delta\mathbf{u}_1 + \frac{\lambda+\mu}{\lambda+2\mu}\,\mathrm{grad}\,\mathrm{div}\,\mathbf{u}_1 - \mathrm{grad}\,\theta = \frac{\partial^2\mathbf{u}_1}{\partial\tau_1^2}\,, \tag{3}$$

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